# Coordinating Multiple Intelligent Reflecting Surfaces Without Channel Information

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Abstract-Conventional beamforming methods for intelligent reflecting surfaces (IRSs) or reconfigurable intelligent surfaces (RISs) typically entail the full channel state information (CSI). However, the computational cost of channel acquisition soars exponentially with the number of IRSs. To bypass this difficulty, we propose a novel strategy called blind beamforming that coordinates multiple IRSs by means of statistics without knowing CSI. Blind beamforming only requires measuring the received signal power at the user terminal for a sequence of randomly generated phase shifts across all IRSs. The main idea is to extract the key statistical quantity for beamforming by exploring only a small portion of the whole solution space of phase shifts. We show that blind beamforming guarantees a signal-to-noise ratio (SNR) boost of  $\Theta(N^{2L})$  under certain conditions, where L is the number of IRSs and N is the number of reflecting elements per IRS. The proposed conditions for achieving the optimal SNR boost of  $\Theta(N^4)$  in a double-IRS system are much easier to satisfy than the existing ones in the literature. Most importantly, the proposed conditions can be extended to a fully general L-IRS system. The above result significantly improves upon the state of the art in the area of multi-IRS-assisted communication. Moreover, blind beamforming is justified via field tests and simulations. In particular, as shown in our field tests at 2.6 GHz, our method yields up to 17 dB SNR boost; to the best of our

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knowledge, this is the first time that the use of multiple IRSs gets verified in the real world.

*Index Terms*—Intelligent reflecting surface (IRS), reconfigurable intelligent surface (RIS), multi-IRS/RIS systems, blind beamforming without channel state information (CSI).

# I. INTRODUCTION

NTELLIGENT reflecting surface (IRS), aka reconfigurable intelligent surface (RIS), is an emerging wireless network device that aims to improve wireless environment by manipulating signal reflections [1], [2], [3], [4]. Owing to its much lower cost and much lower energy consumption, IRS can provide an alternative to small base-station and relay for enhancing throughput, coverage, connectivity, and reliability in future networks such as the industrial Internet of Things (IIoT). While the early studies [1] concentrate on a single IRS, the current trend is towards the multi-IRS coordination [5], [6]. Many existing methods in this field require the full channel state information (CSI), thus suffering the curse of dimensionality when IRSs are deployed extensively. To bypass this difficulty, we propose a novel strategy called *blind beamforming* that is capable of optimizing phase shifts across multiple IRSs in the absence of CSI.

Our approach is inspired by the two recent works [7], [8], which suggest the potential of optimizing phase shifts blindly for a single IRS without CSI. Given the whole solution space  $\Omega$  of phase shifts (which is too large to explore fully), [7], [8] propose only testing a small subset of possible solutions  $S \subset \Omega$  at random, from which a statistical quantity (e.g., the conditional sample mean) of the received signal power can be obtained to help decide phase shifts. The resulting solution is not restricted to S. While [7], [8] focus on a single IRS, this work aims at a full generalization of blind beamforming that accounts for multiple IRSs.

Because the number of channels is exponential in the number of IRSs, channel estimation is a tractable task only in some simple settings, e.g., when there are two IRSs [9], [10], [11], [12], or when the multi-hop reflected channels are all neglected [13]. Some studies are devoted to the overhead reduction for channel estimation in IRS systems, e.g., the deep learning method [14] and the two-timescale optimization [15]. Aside from the computational difficulty, channel estimation for IRS also imposes a huge practical challenge because of the communication chip

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issue as well as the network protocol issue [8]. To the best of our knowledge, the existing prototype realizations of IRS [7], [16], [17], [18], [19], [20] seldom involve channel estimation.

Actually, even if the exact CSI has been provided, it is still quite difficult to decide phase shifts for multiple IRSs. The difficulty arises from the fact that every multi-hop reflected channel is incident to more than one reflecting element (RE) of distinct IRSs and hence their phase shifts must be optimized jointly. To render the problem tractable, a common compromise [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37] is to ignore the multi-hop channels. Many existing analyses and methods build upon this approximation, ranging from delay alignment [21] to ergodic rate [22], secure transmission [23], spectral efficiency [24], outage probability [25], [26], and full-duplex transmission [27]. The above approximation has also been extended to the multiple-user case for a variety of system design problems related to IRS, e.g., the sum rates maximization [28], [29], [30], the IRS placement optimization [31], the target sensing [32], the joint sensing and communication [33], [34], the joint unmanned aerial vehicles (UAV) and IRS aided transmission [35], [36], and the federated learning [37].

However, the above simplified channel model with multiple IRSs could be fundamentally flawed. If each signal reflection is incident to only one IRS, then the multiple IRSs distributed at the different positions can be basically thought of as a single IRS. As a result, the signal-to-noise ratio (SNR) boost is at most  $\Theta(L^2N^2)$  according to [8], where L is the number of IRSs and N is the number of REs of each IRS. In contrast, this work shows that a much higher boost of  $\Theta(N^{2L})$  can be reached by harnessing the multi-hop reflections. Actually, the previous work [38] already shows that the two-hop channels play a crucial role in enabling an SNR boost of  $\Theta(N^4)$  for a double-IRS system. Nevertheless, the argument in [38] is based on a fairly strong assumption that only the two-hop reflections exist while the rest channels are all null. Similarly, [39], [40] only assume the existence of the longest cascaded channels (which are incident to every IRS) from transmitter to receiver in a general L-IRS system. A line of other works [41], [42], [43], [44], [45], [46], [47] simplify the multi-IRS channel model in the opposite way. They only consider the one-hop and the twohop reflections while neglecting all the higher-order reflections. Differing from all the above works, this paper does not require any channels to be zero. As a major result of this work, we show that the highest possible SNR boost of  $\Theta(N^{2L})$  can be achieved by blind beamforming without making any zero approximations of the channels.

The main contributions of this paper are summarized below:

1) We propose a blind beamforming method for the double-IRS system. It extends the existing blind beamforming algorithm by performing the CSM method [8] for the two IRSs sequentially. Although this extension is natural and simple, its performance analysis is by no means trivial. We show that the optimal SNR boost  $\Theta(N^4)$  can be achieved under the following conditions C1 (when multihop channels can be decomposed), C2 (when the phase shift resolution is sufficiently large), and C3 (when the multi-hop channels are sufficiently strong). The proposed optimality conditions are much easier to satisfy than the existing ones in [38], [48].

- 2) We further extend blind beamforming to the fully general case with L IRSs. The extension is still straightforward: we perform CSM [8] sequentially across the L IRSs. Again, the difficulty lies in the performance analysis. The main result in this part of our work is to extend the optimality conditions C1–C3 to the multi-IRS case as D1–D3. To the best of our knowledge, this is the first known set of nontrivial optimality conditions for more than two IRSs.
- 3) Aside from simulations, we conduct field tests to demonstrate the proposed blind beamforming method with multiple IRSs at 2.6 GHz. The test results show that our method can outperform the benchmarks significantly for both indoor and outdoor environments. To the best of our knowledge, this is the first time that the use of multiple IRSs is verified in the real world.

The remainder of the paper is organized as follows. Section II describes the multi-IRS channel model and formulates the beamforming problem mathematically. Section III introduces the blind beamforming method for a double-IRS system. Section IV extends the proposed method to a general *L*-IRS system. Section V shows field test and simulation. Finally, Section VI concludes this work.

The Bachmann-Landau notation is extensively used in the paper: f(n) = O(g(n)) if there exists some c > 0 such that  $|f(n)| \le cg(n)$  for n sufficiently large; f(n) = o(g(n)) if for any c > 0 it holds that |f(n)| < cg(n) for n sufficiently large;  $f(n) = \Omega(g(n))$  if there exists some c > 0 such that  $f(n) \ge cg(n)$  for n sufficiently large;  $f(n) = \Omega(g(n))$  if there exists some c > 0 such that  $f(n) \ge cg(n)$  for n sufficiently large;  $f(n) = \Theta(g(n))$  if f(n) = O(g(n)) and  $f(n) = \Omega(g(n))$  both hold. Moreover, the phase of a complex number  $x \in \mathbb{C}$  is written as [a : b] for two integers a < b. The cardinality of a set  $\mathcal{A}$  is written as  $|\mathcal{A}|$ ; the absolute value of a complex number a is written as |a|. For convenience, we summarize in Table I the main variables used in the sequel.

#### II. SYSTEM MODEL

Consider a point-to-point wireless transmission in aid of  $L \ge 2$  IRSs. Assume that the transmitter and receiver are equipped with one antenna each. Assume also that every<sup>1</sup> IRS consists of N REs. We use  $\ell \in [1:L]$  to index each IRS, and use  $n_{\ell} \in [1:N]$  to index each RE of IRS  $\ell$ . Let  $\theta_{n_{\ell}} \in [0, 2\pi)$  be the phase shift induced by RE  $n_{\ell}$  into its associated reflected channels. From a practical stand [8], [16], [17], [18], [19], [20], assume that each  $\theta_{n_{\ell}}$  of IRS  $\ell$  can only take on values from a uniform discrete set

$$\Phi_{K_{\ell}} = \{\omega, 2\omega, \dots, K_{\ell}\omega\} \text{ where } \omega = \frac{2\pi}{K_{\ell}}$$
(1)

given a positive integer  $K_{\ell} \ge 2$ , namely *discrete beamforming*. We use  $h_{n_1,\dots,n_L}$  to denote the cascaded reflected channel induced by the REs  $(n_1, n_2, \dots, n_L)$ ; let  $n_{\ell} = 0$  if the channel is

<sup>&</sup>lt;sup>1</sup>We assume that IRSs have the same number of REs in order to facilitate performance analysis. But this assumption is not required for the practical use of blind beamforming as discussed in Section V-A.

TABLE I						
LIST OF	MAIN	VARIABLES				

Symbol	Definition
L	number of IRSs
N	number of REs of each IRS
$K_{\ell}$	number of phase shifts choices for each RE of IRS $\ell$
Т	number of random samples for blind beamforming
$n_\ell$	index of the <i>n</i> th RE of IRS $\ell$
$h_{n_1,,n_L}$	cascaded channel induced by REs $n_1, \ldots, n_L$
$u_{n_{\ell}}^{(\ell)}$	factor component of $h_{n_1,,n_L}$ related to RE $n_\ell$
$\theta_{n_\ell}$	phase shift of RE $n_\ell$
$\theta'_{n_\ell}$	solution of $\theta_{n_{\ell}}$ by the proposed method
$\theta_{n_{\ell}}^{\star}$	continuous solution of $\theta_{n_\ell}$ as $K_\ell \to \infty$
$\hat{\theta}^{\star}_{n_{\ell}}$	approximate continuous solution of $\theta_{n_{\ell}}$
$\mathcal{D}_m^{(\ell)}$	set of reflected channels related to RE $m$ of IRS $\ell$
$\mathcal{D}_0^{(\ell)}$	set of channels not related to any RE of IRS $\ell$
$\mathcal{A}_m^{(\ell)}$	subset of $\mathcal{D}_m^{(\ell)}$ unrelated to at least one IRS
$\mathcal{E}_m^{(\ell)}$	subset of $\mathcal{D}_m^{(\ell)}$ related to every IRS

not related to IRS  $\ell$ . For instance, if L = 3 and N = 10, then  $h_{2,0,6}$  represents a reflected channel incident to the 2nd RE of IRS 1 and the 6th RE of IRS 3, which is not related to any RE of IRS 2. In particular,  $h_{0,...,0}$  represents the direct channel from the transmitter to the receiver. For the transmit signal  $X \in \mathbb{C}$  and the complex Gaussian background noise  $Z \sim C\mathcal{N}(0, \sigma^2)$ , the received signal  $Y \in \mathbb{C}$  is given by

$$Y = \sum_{(n_1, \dots, n_L) \in [0:N]^L} h_{n_1, \dots, n_L} e^{j \sum_{\ell=1}^L \theta_{n_\ell}} X + Z.$$
(2)

For each  $n_{\ell} = 0$ , we accordingly set  $\theta_{n_{\ell}} = 0$ . When specialized to the double-IRS case with L = 2, the above equation can be rewritten as

$$Y = \underbrace{h_{0,0}X}_{\text{direct signal}} + \underbrace{\sum_{n_1=1}^{N} h_{n_1,0} e^{j\theta_{n_1}}X}_{\text{reflected signal due to IRS 1}} + \underbrace{\sum_{n_2=1}^{N} h_{0,n_2} e^{j\theta_{n_2}}X}_{\text{reflected signal due to IRS 2}} + \underbrace{\sum_{n_1=1}^{N} \sum_{n_2=1}^{N} h_{n_1,n_2} e^{j(\theta_{n_1} + \theta_{n_2})}X}_{\text{reflected signal due to both IRS 1 & IRS 2}} (3)$$

as illustrated in Fig. 1. In most of this work, we assume a general integer  $L \ge 2$ . Section III focuses on the special case of L = 2. With the transmit power  $P = \mathbb{E}[|X|^2]$ , the received SNR is

$$SNR = \left| \sum_{(n_1, \dots, n_L) \in [0:N]^L} h_{n_1, \dots, n_L} e^{j \sum_{\ell=1}^L \theta_{n_\ell}} \right|^2 \frac{P}{\sigma^2}.$$
 (4)

We wish to evaluate the performance gain brought by the IRSs. Toward this end, let us also compute the SNR without using any IRS as a benchmark, that is

$$SNR_0 = |h_{0,...,0}|^2 \frac{P}{\sigma^2}.$$
 (5)



Fig. 1. A double-IRS system with L = 2, where  $h_{0,0}$  is the direct channel,  $\{h_{n_1,0}, h_{0,n_2}\}$  are the one-hop reflected channels, and  $\{h_{n_1,n_2}\}$  are the two-hop reflected channels.

We seek the optimal set of phase shifts  $\{\theta_{n_{\ell}}\}\$  that maximizes the SNR boost, i.e.,

$$\max_{\substack{\{\theta_{n_s}\}\\ \{\theta_{n_s}\}}} \frac{\text{SNR}}{\text{SNR}_0}$$
(6a)

subject to 
$$\theta_{n_{\ell}} \in \Phi_{K_{\ell}}, \forall n_{\ell}.$$
 (6b)

The difficulties of the above problem are two-fold: (i) the variables are discrete; (ii) the channels  $\{h_{n_1,\ldots,n_L}\}$  are unknown.

# III. DOUBLE-IRS CASE

The conventional paradigm for IRS beamforming comprises two stages: first estimate the cascaded channels  $\{h_{n_1,...,n_L}\}$  and then optimize the phase shifts  $\{\theta_{n_\ell}\}$ . But channel acquisition does not scale well with problem size because the number of channels grows exponentially with the number of IRSs. Alternatively, one may just estimate the channel matrix between every pair of IRSs and subsequently recover the cascaded channels  $\{h_{n_1,...,n_L}\}$  by multiplying the associated between-IRS channel matrices together, so that the number of channels to estimate decreases to  $2NL + {L \choose 2}N^2 = O(N^2L^2)$ . However, the above method is costly in practice because it requires deploying a sensor at each RE to detect the pilot signal for channel estimation. Differing from most approaches in the literature, this work sidesteps channel estimation and optimizes phase shifts directly in the absence of CSI.

## A. Blind Beamforming for a Single IRS

Before proceeding to the double-IRS case, we first review the so-called *conditional sample mean (CSM)* method in [8] for configuring a single IRS without any channel information. We then let L = 1. Since there is only one IRS, the IRS index  $\ell$  can be dropped for ease of notation, i.e.,  $n_{\ell}$  reduces to n.

If all the channels were already known, then a natural idea would be to align each reflected channel  $h_n$  with the direct channel  $h_0$ . If the perfect alignment cannot be achieved due to the discrete constraint  $\Phi_K$ , one may rotate  $h_n$  to the closest possible position to  $h_0$  in the complex plane, namely the *closest point projection (CPP)*, whereby phase shift is determined as

$$\theta_n^{\text{CPP}} = \arg\min_{\theta \in \Phi_K} \left| \theta + \angle h_n - \angle h_0 \right|. \tag{7}$$

The aim of CSM is to mimic CPP without knowing  $\angle h_n$  and  $\angle h_0$ . The method works as follows. We first generate a total

of T random samples  $\boldsymbol{\theta}^{(t)} = \{\theta_n^{(t)} \mid n \in [1:N]\}$  with each  $\theta_n^{(t)}$  drawn uniformly from  $\Phi_K$ , for the sample index  $t \in [1:T]$ . Let  $\mathcal{G}_{n,k} \subseteq [1:T]$  be the set of indices of those samples  $\boldsymbol{\theta}^{(t)}$  satisfying  $\theta_n^{(t)} = k\omega$ , i.e.,

$$\mathcal{G}_{n,k} = \left\{ t \in [1:T] \, \middle| \, \theta_n^{(t)} = k\omega \right\}. \tag{8}$$

We measure the received signal power  $|Y^{(t)}|^2$  corresponding to each random sample  $\theta^{(t)}$ , based on which a conditional sample mean of  $|Y^{(t)}|^2$  is computed for each  $\mathcal{G}_{n,k}$  as

$$\widehat{\mathbb{E}}[|Y|^2 \mid \theta_n = k\omega] = \frac{1}{|\mathcal{G}_{n,k}|} \sum_{t \in \mathcal{G}_{n,k}} |Y^{(t)}|^2.$$
(9)

The solution by CSM, denoted  $\theta'_n$ , maximizes the conditional sample mean with respect to each RE, i.e.,

$$\theta'_n = \arg \max_{\varphi \in \Phi_K} \widehat{\mathbb{E}}[|Y|^2 \,|\, \theta_n = \varphi]. \tag{10}$$

Define the average-reflection-to-direct-signal ratio to be

$$\rho = \frac{\sum_{n=1}^{N} |h_n|^2 / N}{|h_0|^2},\tag{11}$$

which converges in probability to a finite constant for N sufficiently large. The behavior of CSM is characterized by the following proposition.

*Proposition 1:* (Theorem 2 in [8]). The CSM method is equivalent to the CPP method in (7) and yields a quadratic SNR boost in the number of REs in expectation, i.e.,

$$\mathbb{E}\left[\frac{\mathrm{SNR}}{\mathrm{SNR}_0}\right] = \rho \cdot \Theta(N^2), \tag{12}$$

so long as  $K \ge 3$  and  $T = \Omega(N^2(\log N)^3)$ , where the expectation is taken over random samples of  $(\theta_1, \ldots, \theta_N)$ .

*Remark 1:* CSM only requires trying out a polynomial number of possible solutions  $(\theta_1, \ldots, \theta_n)$ , which occupy a small portion of the whole solution space of size  $K^N$ .

*Remark 2:* For the binary beamforming case with K = 2, i.e., when each  $\theta_n \in \{0, \pi\}$ , the SNR boost by CSM may fall below the quadratic. Rather, the SNR boost can be arbitrarily close to 0 dB in the worst-case scenario as shown in [8]. In contrast, an enhanced CSM in [8] maintains the quadratic boost for any  $K \ge 2$ . Nevertheless, the contrived worst-case scenario of CSM rarely occurs in practice, so CSM is still a good choice for binary beamforming.

# B. Blind Beamforming for Two IRSs

We now let L = 2. The above CSM method is extended to the double-IRS case as follows: first optimize IRS 1 while holding IRS 2 fixed, and then optimize IRS 2 while holding IRS 1 fixed, as summarized in Algorithm 1. Simple as the extension looks, it is by no means trivial to analyze its performance. The main result of this subsection is to establish a quartic SNR boost of  $\Theta(N^4)$  under certain conditions.

Let us start with a common misconception. One may think that the SNR boost can be readily verified for the extended CSM because each IRS brings a  $\Theta(N^2)$  boost as shown in Proposition 1 and hence the two IRSs together bring a  $\Theta(N^4)$ 

Algorithm 1 Blind Beamforming for a Double-IRS System

- 1: Initialize all the phase shifts to zero.
- 2: Generate T random samples for IRS 1.
- 3: for t = 1, ..., T do
- 4: Measure the received signal power  $|Y^{(t)}|^2$ .
- 5: end for
- 6: for  $n_1 = 1, ..., N$  do
- 7: **for**  $k = 1, ..., K_1$  **do**
- 8: Compute the conditional sample mean in (9).
- 9: **end for**
- 10: Decide each  $\theta_{n_1}$  for IRS 1 according to (10).
- 11: end for
- 12: Generate T random samples for IRS 2.
- 13: for t = 1, ..., T do
- 14: Measure the received signal power  $|Y^{(t)}|^2$ .
- 15: end for
- 16: for  $n_2 = 1, \ldots, N$  do
- 17: **for**  $k = 1, ..., K_2$  **do**
- 18: Compute the conditional sample mean in (9).
- 19: end for
- 20: Decide each  $\theta_{n_2}$  for IRS 2 according to (10).

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21: end for
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boost. The above argument is problematic in that the boost factor  $\rho$  in (12) associated with IRS 1 is impacted by the later optimization of IRS 2. As a quick example, if the channels between the two IRSs are all zeros so that only  $h_{0,0}$ ,  $h_{n_1,0}$ ,  $h_{0,n_2}$ survive, then the two IRSs can be recognized as one whole IRS, and thus the highest possible boost is  $\Theta(N^2)$ . The reason is that each  $h_{0,n_2}$  is included in the fixed direct channel when analyzing the SNR boost for IRS 1, but subsequently it can be altered dramatically by the optimization of IRS 2. Thus, the key question is: how do we preserve the SNR boost of the previous IRS when optimizing the current IRS? The following theorem provides a set of sufficient conditions in this regard.

*Theorem 1:* If a double-IRS system satisfies the following three conditions:

C1) the channels between the two IRSs are line-of-sight (LoS) so that the two-hop channel matrix has rank one [38] and can be factorized as

$$\begin{bmatrix} h_{1,1} & \cdots & h_{1,N} \\ \vdots & & \vdots \\ h_{N,1} & \cdots & h_{N,N} \end{bmatrix} = \begin{bmatrix} u_1^{(1)} \\ \vdots \\ u_N^{(1)} \end{bmatrix} \begin{bmatrix} u_1^{(2)} & \cdots & u_N^{(2)} \end{bmatrix}, \quad (13)$$

where  $u_n^{(1)} \neq 0$  and  $u_n^{(2)} \neq 0$  for all  $n \in [1:N]$ ; C2)  $K_1, K_2 \geq 3$ ;

C3) there exists a constant  $\gamma \in [0, \frac{\pi}{2} - \frac{\pi}{K_1})$  such that

$$|h_{n_1,0}| \le \sin \gamma \cdot \left| \sum_{n_2=1}^N h_{n_1,n_2} \right|, \ \forall n_1 \in [1:N],$$
 (14)

then the extended CSM method as stated at the beginning of this subsection yields a quartic SNR boost as

$$\mathbb{E}\left[\frac{\mathrm{SNR}}{\mathrm{SNR}_0}\right] = \frac{\delta_1^2 \delta_2^2}{|h_0|^2} \cdot \Theta(N^4)$$
(15)

so long as  $T = \Omega(N^2(\log N)^3)$ , where the expectation is taken over random samples of phase shifts, and

$$\delta_1 \triangleq \frac{1}{N} \sum_{n_1=1}^{N} |u_{n_1}^{(1)}| \quad \text{and} \quad \delta_2 \triangleq \frac{1}{N} \sum_{n_2=1}^{N} |u_{n_2}^{(2)}|.$$
 (16)

*Proof:* Some concrete examples are provided later in Section III-C to illustrate the proposed conditions C1–C3. A formal proof follows. Since  $|h_0|^2$  and P are fixed, it suffices to show that  $\mathbb{E}[|g|^2] = \delta_1^2 \delta_2^2 \cdot \Theta(N^4)$ , where g represents the superposition of all the channels with the IRS phase shifts  $\theta_{n_1}$  and  $\theta_{n_2}$ , i.e.,

$$g(\theta_{n_1}, \theta_{n_2}) = h_{0,0} + \sum_{n_1=1}^{N} h_{n_1,0} e^{j\theta_{n_1}} + \sum_{n_2=1}^{N} h_{0,n_2} e^{j\theta_{n_2}} + \sum_{n_1=1}^{N} \sum_{n_2=1}^{N} h_{n_1,n_2} e^{j(\theta_{n_1}+\theta_{n_2})}.$$
 (17)

To establish  $\mathbb{E}[|g|^2] = \delta_1^2 \delta_2^2 \cdot \Theta(N^4)$ , we need to verify the converse  $\mathbb{E}[|g|^2] = \delta_1^2 \delta_2^2 \cdot O(N^4)$  and the achievability  $\mathbb{E}[|g|^2] = \delta_1^2 \delta_2^2 \cdot \Omega(N^4)$ . The converse is evident since

$$|g|^{2} \leq \left| |h_{0,0}| + \sum_{n_{1}=1}^{N} |h_{n_{1},0}| + \sum_{n_{2}=1}^{N} |h_{0,n_{2}}| + \sum_{n_{1}=1}^{N} \sum_{n_{2}=1}^{N} |h_{n_{1},n_{2}}| \right|^{2} \\ = \delta_{1}^{2} \delta_{2}^{2} \cdot O(N^{4}).$$

The rest of the proof focuses on the achievability.

According to the extended CSM method stated at the beginning of this subsection, we first configure IRS 1 with the phase shifts of IRS 2 fixed to 0, by treating all the channels related to IRS 1 as the reflected channel and the rest as the direct channel. Thus, if  $\theta_{n_1}$  is continuous, its optimal solution is aligning the reflected channel with the direct channel exactly, i.e.,

$$\theta_{n_1}^{\star} = \angle \left( \underbrace{h_{0,0} + \sum_{n_2=1}^N h_{0,n_2}}_{\text{direct channel}} \right) - \angle \left( \underbrace{h_{n_1,0} + \sum_{n_2=1}^N h_{n_1,n_2}}_{\text{reflected channel}} \right).$$

When  $T = \Omega(N^2(\log N)^3)$ , according to Proposition 1, configuring IRS 1 by conditional sample mean is equivalent to rotating the reflected channel to the closest position to the direct channel, i.e.,

$$\theta_{n_1}' = \arg \min_{\theta \in \Phi_{K_1}} \left| \theta - \theta_{n_1}^* \right|. \tag{18}$$

Clearly, we have

$$\left|\theta_{n_1}' - \theta_{n_1}^\star\right| \le \frac{\pi}{K_1}.\tag{19}$$

Moreover, we approximate the continuous solution  $\theta_{n_1}^{\star}$  by removing the single-hop reflect channels, i.e.,

$$\hat{\theta}_{n_1}^{\star} = \angle \left( h_{0,0} + \sum_{n_2=1}^N h_{0,n_2} \right) - \angle \left( \sum_{n_2=1}^N h_{n_1,n_2} \right)$$
$$= \angle \left( h_{0,0} + \sum_{n_2=1}^N h_{0,n_2} \right) - \angle u_{n_1}^{(1)} - \angle \left( \sum_{n_2=1}^N u_{n_2}^{(2)} \right),$$
(20)



Fig. 2. Illustration of the second step in (21). Let  $\boldsymbol{a} = h_{n_1,0}$  and  $\boldsymbol{b} = \sum_{n_2=1}^{N} h_{n_1,n_2}$ . According to (14),  $\boldsymbol{a}$  must lie on a circle with its radius smaller than  $(\sin \gamma) \cdot |\boldsymbol{b}|$ , so the angle between  $\boldsymbol{b}$  and  $\boldsymbol{a} + \boldsymbol{b}$  is no greater than  $\gamma$  as can be seen from the geometry.

where the second equality follows from the assumption in (13). Next, we show that  $\hat{\theta}_{n_1}^{\star}$  can approximate  $\theta'_{n_1}$  closely so long as the conditions C2 and C3 both hold.

With the inequality in (14), we can bound the difference between  $\hat{\theta}_{n_1}^{\star}$  and  $\theta_{n_1}^{\star}$  as

$$\left|\hat{\theta}_{n_{1}}^{\star} - \theta_{n_{1}}^{\star}\right| = \left| \angle \left( h_{n_{1},0} + \sum_{n_{2}=1}^{N} h_{n_{1},n_{2}} \right) - \angle \left( \sum_{n_{2}=1}^{N} h_{n_{1},n_{2}} \right) \right| \le \gamma,$$
(21)

where the second step is explained in Fig. 2. Now, with  $K_1 \ge 3$  in C2 and  $\gamma \in [0, \frac{\pi}{2} - \frac{\pi}{K_1})$  in C3, combining (19) and (21) together gives

$$\left|\hat{\theta}_{n_{1}}^{\star} - \theta_{n_{1}}'\right| \le \gamma + \frac{\pi}{K_{1}} < \frac{\pi}{2}.$$
 (22)

Next, IRS 2 is configured with each phase shift of IRS 1 fixed at  $\theta'_{n_1}$ . We now treat all the channels related to IRS 2 as reflected channel and treat the rest as the direct channel. Thus, if  $\theta_{n_2}$  is continuous, its optimal solution is given by

$$\theta_{n_2}^{\star} = \left( \underbrace{h_{0,0} + \sum_{n_1=1}^{N} h_{n_1,0} e^{j\theta_{n_1}'}}_{\text{direct channel}} \right) - \left( \underbrace{h_{0,n_2} + \sum_{n_1=1}^{N} h_{n_1,n_2} e^{j\theta_{n_1}'}}_{\text{reflected channel}} \right).$$
(23)

Again, when  $T = \Omega(N^2(\log N)^3)$ , Proposition 1 gives

$$\theta_{n_2}' = \arg\min_{\theta \in \Phi_{K_2}} \left| \theta - \theta_{n_2}^{\star} \right| \tag{24}$$

and

$$\left|\theta_{n_2}' - \theta_{n_2}^\star\right| \le \frac{\pi}{K_2}.\tag{25}$$

For ease of notation, we define

$$\xi_{n_2} = h_{0,n_2} + \sum_{n_1=1}^{N} h_{n_1,n_2} e^{j\theta'_{n_1}}.$$
 (26)

It can be shown that

$$|g(\theta_1', \theta_2')|^2 = \left| h_{0,0} + \sum_{n_1=1}^N h_{n_1,0} e^{j\theta_{n_1}'} + \sum_{n_2=1}^N e^{j\theta_{n_2}'} \xi_{n_2} \right|^2$$
$$\geq \left( \cos \frac{\pi}{K_2} \cdot \sum_{n_2=1}^N |\xi_{n_2}| \right)^2, \tag{27}$$

where the last inequality follows by the projection of each  $e^{j\theta'_{n_2}}\xi_{n_2}$  onto  $h_{0,0} + \sum_{n_1=1}^N h_{n_1,0}e^{j\theta'_{n_1}}$  and by the fact that the angle between them is bounded above by  $\pi/K_2$  according to (25). We further bound the  $|\xi_{n_2}|$  as follows:

$$\begin{aligned} \xi_{n_2} | &= \left| h_{0,n_2} + \sum_{n_1=1}^N h_{n_1,n_2} e^{j\theta'_{n_1}} \right| \\ &= \left| \sum_{n_1=1}^N h_{n_1,n_2} e^{j\theta'_{n_1}} \right| + o(N) \\ &= \left| \sum_{n_1=1}^N h_{n_1,n_2} e^{j\theta'_{n_1}} e^{j(\theta'_{n_1} - \hat{\theta}^*_{n_1})} \right| + o(N) \\ &\stackrel{(a)}{=} \left| \sum_{n_1=1}^N u_{n_1}^{(1)} u_{n_2}^{(2)} e^{j(\eta - \angle u_{n_1}^{(1)})} e^{j(\theta'_{n_1} - \hat{\theta}^*_{n_1})} \right| + o(N) \\ &= |u_{n_2}^{(2)}| \cdot \left| \sum_{n_1=1}^N |u_{n_1}^{(1)}| e^{(\theta'_{n_1} - \hat{\theta}^*_{n_1})} \right| + o(N) \\ &\stackrel{(b)}{\geq} |u_{n_2}^{(2)}| \cdot \cos\left(\gamma + \frac{\pi}{K_1}\right) \cdot \sum_{n_1=1}^N |u_{n_1}^{(1)}| + o(N), \end{aligned}$$
(28)

where step (a) uses the shorthand

$$\eta \triangleq \angle \left( h_{0,0} + \sum_{n_2=1}^N h_{0,n_2} \right) - \angle \left( \sum_{n_2=1}^N u_{n_2}^{(2)} \right).$$
(29)

The key step in the above derivation is that  $\eta$  is independent of  $n_1$  and hence can be omitted after step (a). Moreover, step (a) follows by the rank-one assumption in (13) and the definition of  $\hat{\theta}_{n_1}^{\star}$  in (20), and step (b) follows by the bound between  $\theta'_{n_1}$  and  $\hat{\theta}_{n_1}^{\star}$  in (22). Finally, combining (27) and (28) gives

$$|g(\theta_1', \theta_2')|^2 = \Omega\left(\left(\sum_{n_2=1}^N \sum_{n_1=1}^N |u_{n_2}^{(2)}| |u_{n_1}^{(1)}|\right)^2\right)$$
$$= \delta_1^2 \delta_2^2 \Omega(N^4).$$
(30)

The proof of Theorem 1 is then completed.

#### C. Comments on Theorem 1

Although the conditions C1–C3 are not easy to comprehend at first glance, the intuitions behind them turn out to be quite simple. C1 suggests that the channel between any two adjacent IRSs should not be blocked; this makes sense because the two IRSs reduce to a larger single IRS otherwise. C2 suggests that the number of phase shift choices should be more than two; Example 2 will show a worst-case scenario with  $K_1 = K_2 = 2$ in which the IRS gain is arbitrarily close to zero. C3 suggests that the longest reflected channels should be sufficiently strong; this also makes sense because we wish to make sure that the majority of the channels are good, i.e., there are  $N^2$  two-hop channels, while there are only 2N one-hop channels and one direct channel.

Furthermore, we compare the proposed conditions with the existing conditions for the double-IRS system to achieve an SNR boost of  $\Theta(N^4)$ . The conditions in [38] are

- C'1) the matrix factorizing property in C1 holds for all the two-hop reflected channels;
- C'2)  $K_1, K_2 \rightarrow \infty$ , namely the continuous beamforming;
- C'3) the direct channel  $h_{0,0}$  and the one-hop reflected channels  $\{h_{n_1,0}, h_{0,n_2}\}$  all equal zero.

It is easy to see that C'2 is a special case of C2 and that C'3 is a special case of C3. Thus,  $\{C'1-C'3\} \subset \{C1-C3\}$ .

Another set of conditions for the double-IRS system to reach the  $\Theta(N^4)$  SNR boost is proposed in [48] as

- C"1) the matrix factorizing property in C1 holds for all the two-hop reflected channels;
- C"2)  $K_1, K_2 \rightarrow \infty$ , namely the continuous beamforming;
- C"3) the direct channel  $h_{0,0}$  equals zero;
- C"4) the so-called *array response* [48] of IRS 1 remains the same for the IRS-IRS link and the IRS-receiver link; the array response of IRS 2 remains the same for the IRS-IRS link and the transmitter-IRS link; intuitively, this condition holds if IRS 1 is very close to the transmitter while IRS 2 is very close to the receiver.

It turns out that  $\{C1-C3\}$  and  $\{C'1-C''3\}$  are not contained in each other. Nevertheless, the latter only works for the freespace propagation channel model while the former does not assume any particular channel model. Section V will show that the proposed conditions  $\{C1-C3\}$  are much easier to satisfy.

As the final part of this section, we illustrate through some concrete examples why the proposed conditions C1–C3 are critical to the  $\Theta(N^4)$  boost for a double-IRS system.

*Example 1:* (Why is condition C1 needed?). Assume that  $K_1 = K_2 = 4$  and N is an odd number. For any  $n_1$  and  $n_2$ , assume that  $h_{0,0} = h_{n_1,0} = h_{0,n_2} = 0$ ,  $h_{n_1,n_2} = \beta e^{j(n_1+n_2)\pi}$  if  $n_1 \neq n_2$ , and  $h_{n_1,n_2} = \beta e^{j(n_1+n_2)\pi} + 2\beta$  if  $n_1 = n_2$ , where  $\beta > 0$  is a positive constant. This channel setting satisfies C2 and C3 but violates C1 since  $h_{n_1,n_2}h_{n_2,n_1} \neq h_{n_1,n_1}h_{n_2,n_2}$  for  $n_1 \neq n_2$ . It can be shown that the alternating CSM method yields  $\theta'_{n_1} = \theta'_{n_2} = 0$  for every  $n_1$  and  $n_2$  in this case. As a result,  $|\xi_{n_2}| = O(1)$  and thus  $|g(\theta'_{n_1}, \theta'_{n_2})|^2$  is  $O(N^2)$  according to the first line of (27), so the SNR boox is at most quadratic.

Instead, if we let  $h_{n_1,n_2} = \beta e^{j(n_1+n_2)\pi}$  for any  $(n_1,n_2)$  in this example, then C1 is satisfied with  $u_{n_1}^{(1)} = \sqrt{\beta} e^{jn_1\pi}$  and  $u_{n_2}^{(2)} = \sqrt{\beta} e^{jn_2\pi}$ . We have  $\theta'_{n_1} = 0$  if  $n_1$  is odd and  $\theta'_{n_1} = \pi$ otherwise, and  $\theta'_{n_2} = 0$  if  $n_2$  is odd and  $\theta'_{n_2} = \pi$  otherwise. Substituting the above  $(\theta'_{n_1}, \theta'_{n_2})$  in (17) gives a  $\Theta(N^4)$  boost.

*Example 2:* (Why is condition C2 needed?). Assume that  $K_1 = K_2 = 2$  and N is an odd number. For any  $n_1$  and  $n_2$ , assume that  $h_{0,0} = h_{0,n_2} = 0$ ,  $u_{n_1}^{(1)} = \sqrt{\beta}e^{j(n_1+\frac{1}{2})\pi}$ , and  $u_{n_2}^{(2)} = \sqrt{\beta}e^{jn_2\pi}$ , where  $\beta > 0$  is a positive constant. Moreover, let  $h_{n_1,0} = \frac{1}{3}\beta e^{j\frac{\pi}{4}}$  for odd  $n_1$  and let  $h_{n_1,0} = \frac{1}{3}\sqrt{\beta}e^{-j\frac{\pi}{4}}$  for even  $n_1$ . Notice that the above setting satisfies all the conditions in

Algorithm 2 Blind Beamforming for an L-IRS System				
1: Initialize all the $\theta_{n_{\ell}}$ 's to zero.				
2: for $\ell = 1, \ldots, L$ do				
3: Generate T random samples $\{\theta_{n_{\ell}}^{(t)}   n_{\ell} = 1, \dots, N\}.$				
4: <b>for</b> $t = 1,, T$ <b>do</b>				
5: Measure the received signal power $ Y^{(t)} ^2$ .				
6: end for				
7: <b>for</b> $n_{\ell} = 1,, N$ <b>do</b>				
8: for $k = 1, \ldots, K_\ell$ do				
9: Compute the conditional sample mean in (9).				
10: <b>end for</b>				
11: Decide each $\theta_{n_{\ell}}$ for IRS $\ell$ according to (10).				
12: end for				
13: end for				

Theorem 1 except C2. We have  $\theta'_{n_1} = 0$  for all  $n_1$ , and  $\theta'_{n_2} = 0$  if  $n_2$  is odd and  $\theta'_{n_2} = \pi$  otherwise. The resulting SNR boost is  $\Theta(N^2)$ .

In contrast, if  $K_1$  and  $K_2$  are raised to 4 in this example, then we have  $\theta'_{n_1} = -\frac{\pi}{2}$  if  $n_1$  is odd and  $\theta'_{n_1} = \frac{\pi}{2}$  otherwise,  $\theta'_{n_2} = 0$  if  $n_2$  is odd and  $\theta'_{n_2} = \pi$  otherwise. Substituting the above  $(\theta'_{n_1}, \theta'_{n_2})$  in (17) gives a  $\Theta(N^4)$  boost.

*Example 3:* (Why is condition C3 needed?). Assume that  $K_1 = K_2 = 4$  and N is an odd number. For any  $n_1$  and  $n_2$ , assume that  $h_{0,0} = h_{0,n_2} = 0$ ,  $h_{n_1,0} = 2\beta e^{j\frac{\pi}{2}}$ ,  $u_{n_1}^{(1)} = \sqrt{\beta} e^{jn_1\pi}$  and  $u_{n_2}^{(2)} = \sqrt{\beta} e^{jn_2\pi}$ , where  $\beta > 0$  is a positive constant. Observe that  $|h_{n_1,0}| = 2|\sum_{n_2=1}^N h_{n_1,n_2}|$  under the above setting, so C3 in Theorem 1 does not hold here (but C1 and C2 are satisfied). In this case  $\theta'_{n_1} = -\frac{\pi}{2}$  for every  $n_1$ , and  $\theta'_{n_2} = \frac{\pi}{2}$  if  $n_2$  is odd and  $\theta'_{n_2} = -\frac{\pi}{2}$  otherwise. As a result, the SNR boost is  $\Theta(N^2)$  in this case.

In contrast, if  $u_{n_1}^{(1)} = u_{n_2}^{(2)} = \sqrt{\beta}$  for every pair  $(n_1, n_2)$  in this example, then C3 can be satisfied by letting  $\gamma = \frac{\pi}{8}$  when N is sufficiently large. In this case, we have  $\theta'_{n_1} = 0$  for every  $n_1$ , and  $\theta'_{n_2} = \frac{\pi}{2}$  for every  $n_2$ . As a result, the SNR boost of  $\Theta(N^4)$  is achieved.

## IV. GENERAL L-IRS CASE

The CSM method can be further extended to more than two IRSs in a sequential fashion. The initial values of all the  $\theta_{n_{\ell}}$ 's are set to zero. We optimize one IRS at a time while holding the rest IRSs fixed. Algorithm 2 summarizes this sequential CSM method.

Most importantly, the performance bound analysis in Theorem 1 can be carried over to the general  $L \ge 2$  IRSs, as stated in the theorem below.

*Theorem 2:* If an *L*-IRS system satisfies the following three conditions:

D1) there exist a set of nonzero values  $\{u_{n_l}^{(\ell)} \in \mathbb{C} \mid n \in [1 : N], \ell \in [1 : L]\}$  such that each *L*-hop channel (which is related to every IRS) can be decomposed as

$$h_{n_1,\dots,n_L} = \prod_{\ell=1}^L u_{n_\ell}^{(\ell)},\tag{31}$$

where none of  $\{n_1, \ldots, n_L\}$  equals zero;

- D2) the number of phase shift choices satisfy  $K_L \ge 3$  and  $\sum_{\ell=1}^{L-1} \frac{1}{K\ell} < \frac{1}{2}$ ;
- D3) there exists a constant  $\gamma \in [0, \frac{\pi}{L-1}(\frac{1}{2} \sum_{\ell=1}^{L-1} \frac{1}{K_{\ell}}))$  such that

$$\frac{\sum_{(n_1,...,n_L)\in\mathcal{A}_m^{(\ell)}} |h_{n_1,...,n_L}|}{\prod_{i>\ell} |\sum_{n_i=1}^N u_{n_i}^{(i)}| \cdot \prod_{i<\ell} \left[\sum_{n_i=1}^N |u_{n_i}^{(i)}| \cos(\gamma + \frac{\pi}{K_\ell})\right]} \le |u_m^{(\ell)}| \cdot \sin\gamma$$
(32)

for  $\ell \in [1:L-1]$  and  $m \in [1:N]$ , where

$$\mathcal{A}_{m}^{(\ell)} \triangleq \left\{ \left( n_{1}, \dots, n_{L} \right) \middle| n_{\ell} = m, \prod_{\ell=1}^{L} n_{\ell} = 0 \right\}, \quad (33)$$

then Algorithm 2 yields an SNR boost of  $N^{2L}$  as follows:

$$\mathbb{E}\left[\frac{\mathrm{SNR}}{\mathrm{SNR}_0}\right] = \frac{\prod_{\ell=1}^L \delta_\ell^2}{|h_0|^2} \cdot \Theta(N^{2L}),\tag{34}$$

so long as  $T = \Omega \left( N^2 (\log(NL))^3 + N^2 L \log(NL) \right)$  and L = o(N), where the expectation is taken over random samples of phase shifts, and

$$\delta_{\ell} \triangleq \frac{1}{N} \sum_{n_{\ell}=1}^{N} |u_{n_{\ell}}^{(\ell)}|. \ \forall \ell \in [1:L],$$

$$(35)$$

The following lemma plays a key role in proving the above theorem.

Lemma 1: Let  $\theta'_{n_{\ell}}$  be the decision of  $\theta_{n_{\ell}}$  by Algorithm 2, and generalize the definition of  $\hat{\theta}^{\star}_{n_1}$  in (20) as

$$\hat{\theta}_{n_{\ell}}^{\star} \triangleq \angle \left( \sum_{(m_1,\dots,m_L)\in\mathcal{D}_0^{(\ell)}} h_{m_1,\dots,m_L} e^{j\sum_{i=1}^{\ell-1} \theta'_{m_i}} \right) - \angle u_{n_{\ell}}^{(\ell)}$$
$$- \angle \left[ \sum_{(m_1,\dots,m_L)\in\mathcal{E}_{n_{\ell}}^{(\ell)}} \left( e^{j\sum_{i=1}^{\ell-1} \theta'_{m_i}} \prod_{i\neq\ell} u_{m_i}^{(i)} \right) \right], \quad (36)$$

where

$$\mathcal{D}_{m}^{(\ell)} = \{ (m_1, \dots, m_L) | m_\ell = m \}$$
(37)

and

$$\mathcal{E}_m^{(\ell)} = \left\{ (m_1, \dots, m_L) \middle| m_\ell = m, \prod_{i \neq \ell} m_i \neq 0 \right\}.$$
(38)

For any RE  $n_{\ell}$ , we have

$$|\hat{\theta}_{n_{\ell}}^{\star} - \theta_{n_{\ell}}'| \le \gamma + \frac{\pi}{K_{\ell}}$$
(39)

given the constant  $\gamma$  as defined in the condition D3.

*Proof:* See Appendix A.  $\Box$  Equipped with the inequality in (39), we are now ready to prove Theorem 2. The effective channel from the transmitter to

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the receiver is written as a function  $g: \Phi_K^L \mapsto \mathbb{C}$  of the beamforming decision  $(\theta_{n_1}, \ldots, \theta_{n_L})$ , i.e.,

$$g(\theta_{n_1}, \dots, \theta_{n_L}) = \sum_{(n_1, \dots, n_L) \in [0:N]^L} h_{n_1, \dots, n_L} e^{j \sum_{\ell=1}^L \theta_{n_\ell}}.$$
 (40)

To establish an SNR boost of  $\Theta(N^{2L})$ , it suffices to show that  $\mathbb{E}[|g|^2] = \prod_{\ell=1}^L \delta_\ell^2 \cdot \Theta(N^{2L})$ . Again, the converse  $\mathbb{E}[|g|^2] = \prod_{\ell=1}^L \delta_\ell^2 \cdot O(N^{2L})$  is evident, so the rest of this section focuses on the achievability  $\mathbb{E}[|g|^2] = \prod_{\ell=1}^L \delta_\ell^2 \cdot \Omega(N^{2L})$ . Let us first recall how  $\theta_{n_\ell}$  is decided in Algorithm 2. We

Let us first recall how  $\theta_{n_{\ell}}$  is decided in Algorithm 2. We denote by  $\mathcal{D}_{0}^{(\ell)}$  the set of channels not related to any RE of IRS  $\ell$ . When optimizing  $\theta_{n_{\ell}}$ , all the channels in  $\mathcal{D}_{0}^{(\ell)}$  are treated as direct channels, while the rest channels are treated as reflected channels. Recall also that all those IRSs  $i > \ell$  have not yet been configured when optimizing  $\theta_{n_{\ell}}$ , so we have  $\theta_{n_{i}} = 0$  for  $i \in [\ell + 1 : L]$ . The continuous solution of  $\theta_{n_{\ell}}$  is then given by

$$\theta_{n_{\ell}}^{\star} = \angle \left( \sum_{(m_1, \dots, m_L) \in \mathcal{D}_0^{(\ell)}} h_{m_1, \dots, m_L} e^{j \sum_{i=1}^{\ell-1} \theta_{m_i}'} \right) \\ -\angle \left( \sum_{(m_1, \dots, m_L) \in \mathcal{D}_{n_{\ell}}^{(\ell)}} h_{m_1, \dots, m_L} e^{j \sum_{i=1}^{\ell-1} \theta_{m_i}'} \right).$$
(41)

As shown in Appendix B of [8], the probability that  $\theta'_{n_{\ell}} \neq \arg \min_{\theta \in \Phi_{K_{\ell}}} |\theta - \theta^{\star}_{n_{\ell}}|$  for RE  $n_{\ell}$  is bounded above as

$$\mathbb{P}\left\{\theta_{n_{\ell}}' \neq \arg\min_{\theta \in \Phi_{K_{\ell}}} |\theta - \theta_{n_{\ell}}^{\star}|\right\}$$
  
$$\leq \Theta\left(\alpha^{\frac{T}{N^{2}q^{2}}}\right) + \Theta\left(\frac{qN}{T}\right) + \Theta\left(e^{-q/4}\right),$$

where  $\alpha \in (0, 1)$  is a constant related to the channel status and q is a parameter to be specified later. Based on the union of the events bound, the probability that there exists an RE  $n_{\ell}$  in any of the L IRSs such that  $\theta'_{n_{\ell}} \neq \arg \min_{\theta \in \Phi_{K_{\ell}}} |\theta - \theta^{\star}_{n_{\ell}}|$  is bounded above as

$$\mathbb{P}\left\{ \text{at least one } \theta_{n_{\ell}}' \neq \arg\min_{\theta \in \Phi_{K_{\ell}}} |\theta - \theta_{n_{\ell}}^{\star}| \right\}$$
  
$$\leq \sum_{\ell=1}^{L} \sum_{n_{\ell}=1}^{N} \mathbb{P}\left\{ \theta_{n_{\ell}}' \neq \arg\min_{\theta \in \Phi_{K_{\ell}}} |\theta - \theta_{n_{\ell}}^{\star}| \right\}$$
  
$$\leq \Theta\left(NL\alpha^{\frac{T}{N^{2}q^{2}}}\right) + \Theta\left(\frac{qN^{2}L}{T}\right) + \Theta\left(NLe^{-q/4}\right).$$

We can further show that the above error probability is bounded by an o(1) term by letting  $q = \Omega(\log(NL))$  and  $T = \Omega\left(N^2(\log(NL))^3 + N^2L\log(NL)\right)$ . Thus, so long as  $T = \Omega\left(N^2(\log(NL))^3 + N^2L\log(NL)\right)$ , Algorithm 2 would round the above continuous solution to the discrete set  $\Phi_{K\ell}$ :

$$\theta_{n_{\ell}}' = \arg\min_{\theta \in \Phi_{K_{\ell}}} |\theta - \theta_{n_{\ell}}^{\star}|$$
(42)

and hence

$$|\theta_{n_{\ell}}^{\prime} - \theta_{n_{\ell}}^{\star}| \le \frac{\pi}{K_{\ell}}.\tag{43}$$

We then bound the channel strength as follows:

$$\begin{aligned} |g(\theta'_{n_{1}},\ldots,\theta'_{n_{L}})|^{2} \\ &= \left|\sum_{(n_{1},\ldots,n_{L})\in[0:N]^{L}}h_{n_{1},\ldots,n_{L}}e^{j\sum_{i=1}^{L}\theta'_{n_{i}}}\right|^{2} \\ &= \left|\sum_{n_{L}=0}^{N}e^{j\theta'_{n_{L}}}\left(\sum_{(n_{1},\ldots,n_{L-1})}h_{n_{1},\ldots,n_{L}}e^{j\sum_{i=1}^{L-1}\theta'_{n_{i}}}\right)\right|^{2} \\ \stackrel{(a)}{\geq} \left[\sum_{n_{L}=1}^{N}\cos(\theta'_{n_{L}}-\theta^{\star}_{n_{L}})\left|\sum_{(n_{1},\ldots,n_{L-1})}h_{n_{1},\ldots,n_{L}}e^{j\sum_{i=1}^{L-1}\theta'_{n_{i}}}\right|\right]^{2} \\ \stackrel{(b)}{\geq} \left[\sum_{n_{L}=1}^{N}\cos\left(\frac{\pi}{K_{L}}\right)\left|\sum_{(n_{1},\ldots,n_{L-1})}h_{n_{1},\ldots,n_{L}}e^{j\sum_{i=1}^{L-1}\theta'_{n_{i}}}\right|\right]^{2}, \end{aligned}$$
(44)

where step (a) follows by only considering the projection of every  $e^{j\theta'_{n_L}} \sum_{(n_1,\ldots,n_{L-1})} h_{n_1,\ldots,n_L} e^{j\sum_{i=1}^{L-1}\theta'_{n_i}}$  with  $n_L \neq 0$ onto that with  $n_L = 0$ , and step (b) follows by the inequality in (43) directly.

For a fixed  $n_L \neq 0$ , we can further bound the sum component in (44) as follows:

$$\begin{vmatrix} \sum_{(n_1,\dots,n_{L-1})\in[0:N]^{L-1}} h_{n_1,\dots,n_L} e^{j\sum_{i=1}^{L-1} \theta'_{n_i}} \\ -o(N^{L-1}) \end{vmatrix}$$

$$\stackrel{(a)}{=} \begin{vmatrix} \sum_{(n_1,\dots,n_L)\in\mathcal{E}_{n_L}^{(L)}} h_{n_1,\dots,n_L} e^{j\sum_{i=1}^{L-1} \theta'_{n_i}} \\ = \begin{vmatrix} \sum_{(n_1,\dots,n_L)\in\mathcal{E}_{n_L}^{(L)}} \left( u_{n_L}^{(L)} \prod_{\ell=1}^{L-1} u_{n_\ell}^{(\ell)} e^{j\theta'_{n_\ell}} \right) \end{vmatrix}$$

$$\stackrel{(b)}{=} |u_{n_L}^{(L)}| \cdot \left| \sum_{(n_1,\dots,n_L)\in\mathcal{E}_{n_L}^{(L)}} \prod_{\ell=1}^{L-1} |u_{n_\ell}^{(\ell)}| e^{j(\theta'_{n_\ell} - \hat{\theta}^{\star}_{n_\ell})} \right|$$

$$\stackrel{(c)}{=} |u_{n_L}^{(L)}| \sum_{(n_1,\dots,n_L)\in\mathcal{E}_{n_L}^{(L)}} \cos\left[ (L-1)\gamma + \sum_{\ell=1}^{L-1} \frac{\pi}{K_\ell} \right] \prod_{\ell=1}^{L-1} |u_{n_\ell}^{(\ell)}|$$

$$= |u_{n_L}^{(L)}| N^{L-1} \cos\left[ (L-1)\gamma + \sum_{\ell=1}^{L-1} \frac{\pi}{K_\ell} \right] \prod_{i=1}^{L-1} \delta_i, \quad (45)$$

where step (a) follows as the number of  $(n_1, \ldots, n_{L-1})$ 's with at least one  $n_i = 0$  equals  $(N+1)^{L-1} - N^{L-1} = o(N^{L-1})$ , step (b) follows by (36), and step (c) follows by (39).

Substituting the lower bound (45) in (44), we obtain

$$g(\theta'_{n_1},\ldots,\theta'_{n_L})|^2 = \left(\prod_{\ell=1}^L \delta_\ell^2\right) \cdot \Omega(N^{2L}).$$
(46)

Furthermore, combining the above result with the evident fact that  $\mathbb{E}[|g|^2] = \prod_{\ell=1}^L \delta_\ell^2 \cdot O(N^{2L})$  leads us to the boost  $\Theta(N^{2L})$ . The proof of Theorem 2 is thus completed.



(a) controller-centered type



(b) receiver-centered type

Fig. 3. Two different paradigms of blind beamforming.

We cannot provide a performance guarantee for the blind beamforming method if the proposed conditions are not satisfied. However, we clarify that the proposed conditions are already much easier to satisfy than the existing ones. Besides, our field tests in the next section show that the proposed blind beamforming method still yields excellent performance in the real world even if the conditions are not satisfied. Most importantly, the insights and intuitions behind our theoretical conditions can guide the IRS deployment in practice, so it is worthwhile to expand the existing conditions even if the new conditions still cannot be strictly satisfied.

Theorem 2 implies that only one round of configuration (i.e., every IRS is optimized one time regardless of L) suffices to attain an SNR boost of  $\Theta(N^{2L})$ . This is of practical significance when the IRSs are extensively deployed in the network. We also clarify that the conditions D1–D3 are *sufficient but not necessary* for the  $\Theta(N^{2L})$  SNR boost.

We further discuss how the sequential CSM algorithm can be carried out in practice. There are four parties—transmitter, receiver, IRSs, and an IRS controller in the protocol; there is a feedback link from the receiver to the IRS controller; there is a control link from IRS controller to each IRS. A codebook of prescribed pseudo-random samples of phase shift arrays is generated beforehand. There are two possible paradigms of blind beamforming as illustrated in Fig. 3, which are the controllercentered type and the receiver-centered type.

For the controller-centered paradigm, the codebook is only revealed to the IRS controller. Let us consider the  $\ell$ th iteration of blind beamforming. First, the IRS controller activates IRS  $\ell$ 

through the control link, and then sets its phase shift array to the samples in the codebook sequentially. In the meanwhile, for each sample, the receiver reports the received signal power to the IRS controller through the feedback link. After completing T samples, the IRS controller uses the power feedback from the receiver so far to perform CSM. Finally, the IRS controller configures IRS  $\ell$  as the CSM output through the control link.

Alternatively, by the receiver-centered paradigm, the codebook is revealed to the receiver in addition. The sampling part follows the controller-centered case, but the receiver now records its measured received signal power with respect to each sample, instead of reporting it back to the IRS controller. After T samples, the receiver performs CSM based on the codebook and the received signal power data, and then reports the CSM output to the IRS controller through the feedback link. Finally, the IRS controller configures IRS  $\ell$  according to the feedback from the receiver.

*Remark 3:* (Same Phase-Shift Resolution). If the L IRSs have the same phase-shift resolution, i.e., when all the  $K_{\ell}$  values are equal, then D2 in Theorem 2 reduces to every  $K_{\ell} > 2(L-1)$ .

Remark 4: (Overhead Cost). For the double-IRS case, our theoretical lower bound on T with provable performance is  $T = \Omega \left( N^2 (\log N)^3 \right)$  as stated in Theorem 1; but the practical choice of T can be much smaller as shown in Fig. 11. By contrast, since there are  $N^2$  unknown channels, it requires at least  $N^2$  pilot symbols to solve the channel estimation equation [10], [12]. (The work in [49] argues that 2N pilot symbols are sufficient for channel estimation for some special cases.) We will explicitly compare blind beamforming with the channelestimation-based methods [12], [49] in Section V-B.

*Remark 5:* (Computational Complexity). For each IRS, it requires O(NT) to compute all the conditional sample mean values, and it further requires O(NK) to decide phase shifts for the current IRS based on the conditional sample mean. For a total of L IRSs, the overall computational complexity of blind beamforming equals O(LN(K + T)).

#### V. EXPERIMENTS

# A. Field Tests

Throughout our field tests, the transmit power is fixed at -5 dBm and the carrier frequency is 2.6 GHz. The following three IRSs are used:

- IRS 1 with 294 REs and 2 phase shift choices {0, π} for each RE, i.e., N = 294 and K<sub>1</sub> = 2;
- IRS 2 also with N = 294 and  $K_2 = 2$ ;
- IRS 3 with N = 64 and  $K_3 = 4$ .

Notice that the condition C2 or D2 is not satisfied in our field test. Actually, the existing IRS prototypes mostly adopt K = 2 or K = 4 because of the hardware cost and the system robustness [16], [19], [50], [51]. Notice also that we do not always assume that all the IRSs have the same values of N as in the theoretical model in Section II. The following methods are compared:

- Without IRS: IRS is not used.
- Zero Phase Shifts: Fix all phase shifts to be zero.



Fig. 4. Field test with two IRSs deployed in an indoor hallway.



Fig. 5. Layout drawing of the indoor field test. The two IRSs are placed in two corners for most methods, but are merged into a single larger IRS placed in the middle for "Physical Single-IRS" as indicated by the dashed lines.



Fig. 6. Outdoor field test with three IRSs deployed alongside an open café.

- *Random Beamforming:* Try out  $L \times 1000$  random samples of phase shift vectors and choose the best.
- *Virtual Single-IRS:* Ignore the multi-hop channels and treat multiple IRSs as a single one; optimize phase shifts by the method in [8] with  $L \times 1000$  random samples.
- *Physical Single-IRS:* Put multiple IRSs together at the same position to form a single larger IRS; optimize phase shifts by the method in [8] with  $L \times 1000$  random samples.
- *Proposed Blind Beamforming:* Coordinate multiple IRSs by Algorithm 2 that uses 1000 random samples per IRS.

The SNR boost is evaluated by taking "without IRS" as baseline. We consider the following two transmission scenarios:

- *Indoor Environment:* Deploy IRS 1 and IRS 2 in a U-shaped hallway inside an office building as shown in Fig. 4. The testbed layout is specified in Fig. 5. The transmission is blocked by the walls.
- *Outdoor Environment:* Deploy three IRSs alongside an open café as shown in Fig. 6. The testbed layout is specified in Fig. 7. The transmission is occasionally blocked by the crowd and also suffers interference which is treated as noise.



Fig. 7. Layout drawing of the outdoor field test. For Physical Single-IRS, we move IRS 1 and IRS 2 to the positions indicated by the dashed lines.

TABLE II SNR BOOSTS ACHIEVED BY THE DIFFERENT METHODS

	SNR Boost (dB)		
Method	Indoor	Outdoor	
Zero Phase Shifts	2.74	2.91	
Random Beamforming	5.33	8.48	
Virtual Single-IRS	12.07	10.80	
Physical Single-IRS	3.31	7.06	
Blind Beamforming	17.08	14.09	

Table II summarizes the SNR boost performance of the different methods. As shown in the row of Zero Phase Shifts, placing IRSs in the environment (either indoor or outdoor) can already increase SNR by more than 2 dB even without any optimization. Then a simple heuristic optimization method such as Random Beamforming can reap a higher SNR gain. Observe also that Virtual Single-IRS outperforms the above methods significantly, e.g., it improves upon Random Beamforming by around 7 dB for the indoor case.

In contrast, the proposed Blind Beamforming enhances SNR further even though C2 or D2 is not satisfied, e.g., its SNR boost is about 5 dB higher than that of Virtual Single-IRS for the indoor case, and about 3 dB higher for the outdoor case. This further gain is due to the capability of Blind Beamforming to take those multi-hop reflections into account. For this reason, the advantage of Blind Beamforming over Virtual Single-IRS is greater for the indoor case in which the multi-hop reflections play a key role. Another interesting fact from Table II is that Physical Single-IRS is much worse than the other methods especially in the indoor environment. Although its phase shifts have been carefully optimized by the method in [8], its performance is still limited by the deficiency of multi-hop reflections.

#### B. Simulation Tests

We now validate the performance of the proposed blind beamforming algorithm in simulations which can admit many more REs at each IRS and many more IRSs. The channels are generated as follows. We refer to the transmitter as node 0, a total of L IRSs as nodes 1 through L, and the receiver as node L + 1. We denote by  $d_{ij}$  (in meters) the distance between node *i* and node *j*. The pathloss model follows previous works [1], [8], [52]. If there is LoS propagation between node *i* and node *j* then the corresponding pathloss is computed as

$$PL_{ii} = 10^{-(30+22\log_{10}(d_{ij}))/20}.$$
(47)

Otherwise, i.e., when the channel between node i and node j is non-line-of-sight (NLoS), the pathloss is computed as

$$PL_{ij} = 10^{-(32.6 + 36.7 \log_{10}(d_{ij}))/20}.$$
(48)

Following [40], [53], we assume that REs are arranged as a uniform linear array with spacing  $\xi = 0.03$  meters at each IRS. We let the signal wavelength be  $\lambda = 0.06$  meters. Moreover, denote by  $\vartheta_{i,j}$  the angle of departure (AoD) from node *i* to node *j*, and  $\psi_{i,j}$  the angle of arrival (AoA) from node *j* to node *i*. For the LoS case, the channel from the transmitter to RE  $n_{\ell}$  is given by

$$g_{0,n_{\ell}} = \sqrt{\mathrm{PL}_{0,\ell}} \cdot e^{-j\frac{2\pi}{\lambda}d_{0,\ell}} \cdot e^{-j\frac{2\pi}{\lambda}\xi(n_{\ell}-1)\cos\psi_{\ell,0}}, \qquad (49)$$

the channel from RE  $n_{\ell}$  to the receiver is given by

$$g_{n_{\ell},L+1} = \sqrt{\mathrm{PL}_{\ell,L+1}} \cdot e^{-j\frac{2\pi}{\lambda}d_{\ell,L+1}} \cdot e^{-j\frac{2\pi}{\lambda}\xi(n_{\ell}-1)\cos\vartheta_{\ell,L+1}},$$
(50)

and the channel from RE  $n_{\ell}$  to RE  $n_{\ell'}$ ,  $\ell \neq \ell'$ , is given by

$$g_{n_{\ell},n_{\ell'}} = \sqrt{\mathrm{PL}_{\ell,\ell'}} \cdot e^{-j\frac{2\pi}{\lambda}d_{\ell,\ell'}} \cdot e^{-j\frac{2\pi}{\lambda}\xi(n_{\ell}-1)\cos\vartheta_{\ell,\ell'}} \\ \cdot e^{-j\frac{2\pi}{\lambda}\xi(n_{\ell'}-1)\cos\psi_{\ell',\ell}}.$$
 (51)

For the NLoS case, we generate channels as

0

$$g_{0,n_{\ell}} = \sqrt{\mathrm{PL}_{0,\ell}} \cdot \zeta_{0,n_{\ell}},\tag{52}$$

$$g_{n_{\ell},L+1} = \sqrt{\mathsf{PL}_{\ell,L+1}} \cdot \zeta_{\ell,L+1},\tag{53}$$

$$g_{n_{\ell},n_{\ell'}} = \sqrt{\mathrm{PL}_{\ell,\ell'}} \cdot \zeta_{n_{\ell},n_{\ell'}},\tag{54}$$

where  $\zeta_{0,n_{\ell}}, \zeta_{\ell,L+1}, \zeta_{n_{\ell},n_{\ell'}}$  are drawn i.i.d. from the complex Gaussian distribution  $\mathcal{CN}(0,1)$ . For both LoS and NLoS cases, each multi-hop channel  $h_{n_1,\dots,n_L}$  can be obtained by multiplying together a subset of channels in  $\{g_{0,n_{\ell}}, g_{n_{\ell},L+1}, g_{n_{\ell},n_{\ell'}}\}$ . Moreover, the transmit power equals 30 dBm, the background noise power equals -98 dBm, and all the  $K_{\ell}$ 's are fixed to be 4 throughout the simulation tests, i.e.,  $\Phi_{K_{\ell}} = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$  for all  $\ell \in [1:L]$ .

1) Double-IRS System: We begin with a double-IRS system as shown in Fig. 8. Assume that the two IRSs have 100 REs each and that T = 1000 random samples are taken for each IRS. Assume that  $\vartheta_{n_1,n_2} = \vartheta_{n_2,L+1} = \psi_{0,n_1} = \psi_{n_1,n_2} = 5.6^{\circ}$ . Fig. 9 compares the performance of our blind beamforming scheme with the channel-estimation-based methods in [12], [49]. Two different channel scenarios are considered here: Fig. 9(a) assumes that only the two-hop reflected channels are LoS while the rest channels are NLoS; Fig. 9(b) assumes that most channels are LoS except the direct channel. It can be seen that the proposed blind beamforming method outperforms the channelestimation-based methods in the first scenario; however, when more channels become LoS as in the second scenario, the channel-estimation-based methods perform better. The reason is that if most channels are NLoS then they are too weak to be estimated accurately.

Also, these state-of-the-art channel estimation methods require setting each phase shift according to the DFT matrix, but



Fig. 8. The double-IRS system considered in our simulations. The position coordinates are in meters.



(a) when only the two-hop reflected channels are LoS



(b) when only the direct channel is NLoS

Fig. 9. Blind beamforming versus channel-estimation-based methods for a double-IRS system.

this can violate the discrete phase shift constraint, e.g., they do not work for the case where each phase shift is either 0 or  $\pi$ . Besides, it is difficult to extend these channel estimation methods to account for more than two IRSs.

Moreover, if T falls below the above bound, our blind beamforming method still works only that its performance cannot be guaranteed by our current theory anymore. However, if there are fewer than  $N^2$  pilot symbols, then the linear system for channel estimation cannot be solved, so the channel estimation cannot be performed at all (unless under some certain sparsity assumptions). In our field tests, letting T = 1000L can already yield good performance for blind beamforming with N = 294, so the corresponding overhead is  $1000 \times 2 \times 2 = 4000$  symbols, while the estimation methods in [10], [12] require at least  $294 \times 294 \approx 860000$  symbols.

Last, from a practical implementation point of view, the channel estimation methods require reading the received symbol  $Y \in \mathbb{C}$  from the communication chip of the receiver device. By contrast, blind beamforming only requires the received signal power information, which needs a lower level of authority and hence is much easier to implement.

Fig. 10 compares the blind beamforming method with the position-based methods [38], [48]. Again two scenarios are considered: Fig. 10(a) assumes that only the two-hop reflected channels are LoS while Fig. 10(b) assumes that only the direct channel is NLoS. In the first scenario, we see that blind beamforming is close to the position-based method in [38], but it is much worse than the method in [48]. Nevertheless, when it comes to the second scenario, blind beamforming outperforms the benchmark methods significantly. It is worth pointing out that these position-based methods [38], [48] heavily rely on the *free-space line-of-sight propagation* channel model, while our condition does not assume any particular channel model.

Moreover, Fig. 11 compares the growth curves of the SNR boost with the IRS size N under the different values of T. Observe that the growth curve is approximately quartic when  $T = \Theta(N^2(\log N)^3)$ ; this result agrees with Theorem 1. Observe also that the SNR boost still grows rapidly with N even if T falls below the lower bound in Theorem 1, e.g., when  $T = \Theta(N^2)$ .

2) General L-IRS System: We now add more IRSs to the system. Consider a  $100 \times 100 \text{ m}^2$  2D square area as shown in Fig. 12. The transmitter is located at (5,5) and the receiver is located at (95,95). There are 8 possible positions to deploy IRSs: (19.5,77.3), (25.8,15.8), (29.9,11.5), (34.1,60.7), (43.8,15.0), (61.5,84.1), (71.8,35.5) and (73.6,76.2), all in meters. Thus, including the transmitter and receiver, there are a total of 10 nodes in our case. Following [40], the propagation status between any nodes are randomly set to LoS with 60% probability and to NLoS with 40% probability. The realization of the propagation statuses in our case can be expressed in an adjacency matrix as

A =	0	0	1	0	0	0	1	0	0	0	
	0	0	1	0	1	1	0	0	0	1	
	1	1	0	1	0	1	0	1	0	0	
	0	0	1	0	1	1	1	0	0	1	
	0	1	0	1	0	1	1	1	0	1	
	0	1	1	1	1	0	1	1	0	1	1
	1	0	0	1	1	1	0	1	1	0	
	0	0	1	0	1	1	1	0	1	0	
	0	0	0	0	0	0	1	1	0	1	
	0	1	0	1	1	1	0	0	1	0	
	_										



(a) when only the two-hop reflected channels are LoS



Fig. 10. Blind beamforming versus position-based methods for a double-IRS system.



Fig. 11. SNR boost versus number of REs under the different T settings.



Fig. 12. The IRS routing when L = 8 and N = 600.



Fig. 13. The IRS routing when L = 8 and N = 1200.

where the entry  $A_{ij}$  equals 1 if the channel between node i and node j is LoS, and 0 otherwise.

As suggested in [40], only a subset of the possible positions are selected for the IRS placement, namely the IRS routing. For instance, Fig. 12 shows the IRS placement when N = 600, and Fig. 13 shows the IRS placement when N = 1200, both based on the graph-theoretical algorithm in [40]. Observe that more IRSs are used for routing when the number of REs per IRS N is increased. The rationale is that we have a tradeoff when deciding how many IRSs to use. Clearly, using more IRSs gives more multi-hop reflected paths; in order to reap the multipath gain, we need to orient the reflection beam of each IRS toward the subsequent IRS, but this requires high-resolution beamforming and thus N should be sufficiently large. On the other hand, using too many IRSs would increase the reflection loss. Due to the above two factors, the number of IRSs should be neither too large nor too small. Now, if N becomes larger, then the first factor outweighs, so it encourages using more IRSs. Actually, this tradeoff has already been pointed out in the previous work [40].



Fig. 14. SNR boost versus number of REs for the different algorithms.

For the comparison purpose, we consider the following two beamforming methods as benchmarks:

- *CPP with Perfect CSI*: Assume that the precise channel information is already known. Then perform CPP across the IRSs sequentially.
- *Mei-Zhang Method with Perfect CSI* [40]: First perform the continuous beamforming algorithm in [40] and then round the solution to the discrete set  $\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$ . This method assumes that perfect CSI is available.

Notice that the above two competitor methods both require perfect CSI. However, to the best of our knowledge, there is yet no effective way of estimating channels for more than 2 IRSs in the existing literature.

Fig. 14 compares the proposed Algorithm 2 with the benchmarks under the different settings of N. For Algorithm 2, we let  $T = 20 \times N$ . It can be seen that the proposed algorithm is fairly close to CPP. Actually, the proposed algorithm would approach CPP when  $T \rightarrow \infty$ ; the former can be thought of as a practical implementation of the latter. Mei-Zhang method is about 2 dB higher than the proposed algorithm when N = 400. But when N is raised to 600, the proposed algorithm starts to overtake, and their gap becomes larger when N is further increased. Again, not requiring CSI is a distinct advantage of the proposed algorithm as compared to these benchmarks.

Finally, we evaluate how likely the proposed conditions C1-C3 and D1-D3 are satisfied when the IRSs are randomly placed and the channels are randomly generated. We also consider the existing conditions C'1–C'3 in [38] and C"1–C"4 in [48] (but they only account for the double-IRS case). As shown in Fig. 12, the transmitter and the receiver are located at (5, 5) and (95, 95), respectively. The coordinate vector  $(x_{\ell}, y_{\ell})$  of each IRS  $\ell$  is uniformly distributed in  $[5 + \frac{90(\ell-1)}{L}, 5 + \frac{90\ell}{L}] \times [5 + \frac{90(\ell-1)}{L}, 5 + \frac{90\ell}{L}]$ . Following the indoor field test in Fig. 5, we assume that the IRSs are well placed so that the channels between two adjacent IRSs as well as the transmitter-to-the-first-IRS channel and the last-IRS-to-receiver channel are all LoS. For the rest channels, assume that each is LoS with probability  $\eta$ . Following [38], [48], we assume that the channel equals zero



Fig. 15. Probability of the conditions for achieving an SNR boost of  $\Theta(N^{2L})$  to be satisfied, where  $\eta$  is the probability of each channel being LoS.

whenever it is NLoS. Let N = 100 and let each  $K_{\ell} = 2L$ . In particular, we claim that condition C"4 in [48] is satisfied if the approximation gap is less than 10% of the norm of the original array response.

Fig. 15 shows how likely the conditions C1–C3, D1–D3, C'1-C'3 and C"1-C"4 are satisfied over 200 tests when the IRSs are randomly placed in the network and the channels are randomly generated; different values of the parameter  $\eta$  are considered here. This figure shows that our conditions C1-C3 are much more likely to be satisfied than the existing conditions in the literature. The conditions D1-D3 are less likely to be satisfied since they account for more than 2 IRSs in this case. In general, it is increasingly difficult to satisfy the conditions D1–D3 as more and more IRSs are deployed. In particular, observe that the conditions C"1-C"4 in [48] are extremely difficult to meet; the probability of satisfying C"1-C"4 is almost zero in our case. The reason is that C"1-C"4 can be roughly interpreted as requiring IRS 1 (resp. IRS 2) to be very close to the transmitter (resp. receiver) but such conditions are rarely met in our random network. Also, the conditions C'1-C'3 are less likely to be satisfied than C1–C3 because they require many channels to be zero.

# VI. CONCLUSION

This work proposes a statistical approach to the multi-IRS beamforming problem in the absence of channel information. For a general *L*-IRS-assisted wireless transmission, we show that the proposed blind beamforming algorithm guarantees an SNR boost of  $\Theta(N^{2L})$ —which is the highest possible SNR boost obtained from *L* IRSs, under some certain conditions. This blind beamforming strategy has two major advantages over the existing methods. First, it does not entail any channel estimation and yet can yield provable performance. Second, its optimality condition is far less strict than the existing one in [40], e.g., those short reflected channels need not be zero for blind beamforming to reach the  $\Theta(N^{2L})$  boost. Remarkably, as

shown in the real-world experiments at 2.6 GHz, blind beamforming for multiple IRSs increases SNR by over 17 dB in the hallway of an office building, and by over 14 dB near an open café. Moreover, simulations show that much higher gain can be reaped by blind beamforming when the IRSs become larger in size or when more IRSs are deployed.

# APPENDIX A: PROOF OF LEMMA 1

We establish Lemma 1 by induction. When  $\ell = 1$ , we have

$$\frac{\sum_{(n_1,\dots,n_L)\in\mathcal{A}_{n_1}^{(1)}}h_{n_1,\dots,n_L}|}{\sum_{(n_1,\dots,n_L)\in\mathcal{E}_{n_1}^{(1)}}h_{n_1,\dots,n_L}|} = \frac{\left|\sum_{(n_1,\dots,n_L)\in\mathcal{A}_{n_1}^{(1)}}h_{n_1,\dots,n_L}\right|}{\prod_{\ell=2}^L \left|\sum_{n_\ell=1}^N u_{n_\ell}^{(\ell)}\right| \cdot \left|u_{n_1}^{(1)}\right|} \\ \leq \frac{\sum_{(n_1,\dots,n_L)\in\mathcal{A}_{n_1}^{(1)}}\left|h_{n_1,\dots,n_L}\right|}{\prod_{\ell=2}^L \left|\sum_{n_\ell=1}^N u_{n_\ell}^{(\ell)}\right| \cdot \left|u_{n_1}^{(1)}\right|} \\ \leq \sin\gamma, \tag{55}$$

where the last inequality follows by the condition D3 stated in Theorem 2. Now, with a and b in Fig. 2 redefined as  $\sum_{(n_1,\ldots,n_L)\in \mathcal{A}_{n_1}^{(1)}} h_{n_1,\ldots,n_L}$  and  $\sum_{(n_1,\ldots,n_L)\in \mathcal{E}_{n_1}^{(1)}} h_{n_1,\ldots,n_L}$  respectively, we have

$$\left| \angle \left( \sum_{(n_1,\dots,n_L)\in\mathcal{A}_{n_1}^{(1)}} h_{n_1,\dots,n_L} + \sum_{(n_1,\dots,n_L)\in\mathcal{E}_{n_1}^{(1)}} h_{n_1,\dots,n_L} \right) - \angle \left( \sum_{(n_1,\dots,n_L)\in\mathcal{E}_{n_1}^{(1)}} h_{n_1,\dots,n_L} \right) \right| \le \gamma,$$
(56)

which can be recognized as

$$|\hat{\theta}_{n_1}^{\star} - \theta_{n_1}^{\star}| \le \gamma.$$
(57)

Combining the above inequality with (43) yields

$$\hat{\theta}_{n_1}^{\star} - \theta_{n_1}' | \le \gamma + \frac{\pi}{K_1}.$$
(58)

Thus, Lemma 1 is verified for  $\ell = 1$ .

Assuming that Lemma 1 holds for all  $\ell < i$ , we now proceed to the case of  $\ell = i$ . It can be shown that

-1

$$\frac{\left|\sum_{(n_{1},...,n_{L})\in\mathcal{A}_{n_{i}}^{(i)}}h_{n_{1},...,n_{L}}e^{j\sum_{s=1}^{i}\theta_{n_{s}}}\right|}{\left|\sum_{(n_{1},...,n_{L})\in\mathcal{E}_{n_{i}}^{(i)}}h_{n_{1},...,n_{L}}e^{j\sum_{s=1}^{i-1}\theta_{n_{s}}'}\right|} \\ \stackrel{(a)}{=} \frac{\left|\sum_{(n_{1},...,n_{L})\in\mathcal{A}_{n_{i}}^{(i)}}h_{n_{1},...,n_{L}}e^{j\sum_{s=1}^{i-1}\theta_{n_{s}}'}\right|}{\left|\sum_{(n_{1},...,n_{L})\in\mathcal{E}_{n_{i}}^{(i)}}\prod_{si}u_{n_{s}}^{(s)}\cdot u_{n_{i}}^{(i)}\right|}} \\ \leq \frac{\sum_{(n_{1},...,n_{L})\in\mathcal{A}_{n_{i}}^{(i)}}\left|h_{n_{1},...,n_{L}}\right|}{\left|\sum_{(n_{1},...,n_{L})\in\mathcal{E}_{n_{i}}^{(i)}}\prod_{si}u_{n_{s}}^{(s)}\cdot u_{n_{i}}^{(i)}\right|}} \\ \stackrel{(b)}{\leq} \frac{\sum_{(n_{1},...,n_{L})\in\mathcal{E}_{n_{i}}^{(i)}}\left|\sum_{n_{s}=1}^{N}u_{n_{s}}^{(s)}\right|\cdot\prod_{s$$

where step (a) follows by the condition D1 in Theorem 2, step (b) follows since (39) is assumed to hold for all  $\ell < i$ , and step

(c) follows by the condition D3 in Theorem 2. Repeating the former steps (56)–(58), we arrive at

$$|\hat{\theta}_{n_i}^{\star} - \theta_{n_i}'| \le \gamma + \frac{\pi}{K_i}.$$
(60)

The proof is then completed.

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